Technical Comments

AIAA 82-4056

Comment on "Critical Field Length Calculations for Preliminary Design"

Andrzej Wortman*
ISTAR Inc., Santa Monica, Calif.

IN Ref. 1, Powers presents preliminary design calculations of aircraft takeoff performance by integrating the equation of motion under the assumptions of constant aerodynamic coefficients. Actually, aircraft aerodynamic coefficients vary considerably as the Reynolds number, attitude and proximity of the wing to the ground change with speed. The prime purpose of this Comment is to point out that the time and distance at liftoff are quite sensitive to variations of aerodynamic coefficients so that the simplified procedure proposed by Powers is not suitable for preliminary design calculations, but is at best, a very crude estimate. The secondary purpose here is to present a sensitivity analysis which may be used to ascertain the bounds on errors due to mathematically convenient but unrealistic basic assumptions.

Analysis

The present nomenclature generally follows that of Ref. 1 and the acceleration $\mathrm{d}V/\mathrm{d}t$ of an aircraft of mass m is expressed as

$$m\frac{\mathrm{d}V}{\mathrm{d}t}=T-R-\frac{\rho C_D A}{2}V^2 \tag{1}$$

with

$$R=\mu\left(W-\frac{\rho C_L A}{2}V^2\right) \tag{2}$$

and μ is the coefficient of rolling resistance. If, as was done in Ref. 1, the extreme assumption of constant C_L and C_D is made, then integration of Eq. (1) in terms of time t or distance $\mathrm{d}s=V\mathrm{d}t$ requires only elementary integrals, and the results are

$$\tau=\tfrac{1}{2}\ln(1+Z/1-Z) \tag{3}$$

$$\sigma=-\tfrac{1}{2}\ln(1-Z^2) \tag{4}$$

with

$$\tau=(tF/mV_0)\phi^{1/2} \tag{5a}$$

$$\sigma=sF/mV_0^2 \tag{5b}$$

$$Z=(V/V_0)\phi^{-1/2} \tag{5c}$$

$$\phi=(T-\mu W)/F \tag{5d}$$

$$F=\rho(C_D-\mu C_L)AV_0^2/2 \tag{5e}$$

Received July 30, 1981.

*Technical Director. Member AIAA.

The reference speed V_0 is taken to be the liftoff speed. Aerodynamic coefficients are contained in F so that sensitivity relations are obtained through logarithmic differentiation which yields

$$\frac{\mathrm{d}t}{t}=G\frac{\mathrm{d}F}{F} \tag{6a}$$

$$G=0.5+Z\Big/\left[(1-Z^2)\ln\left(\frac{1+Z}{1-Z}\right)\right] \tag{6b}$$

$$\frac{\mathrm{d}s}{s}=-H\frac{\mathrm{d}F}{F} \tag{7a}$$

$$H=1+Z^2/[(1-Z^2)\ln(1-Z^2)] \tag{7b}$$

Both G and H are functions of Z and are readily estimated for a wide range of aircraft.

Estimates of Z

It is convenient to take

$$T=rW \tag{8}$$

$$C_D=C_{D,0}+C_{D,f}+C_{D,i} \tag{9}$$

with $C_{D,0}$ being all the drag coefficient contributions which are not accounted for by the viscous and induced drag coefficients $C_{D,f}$ and $C_{D,i}$ respectively. Taking the latter as $C_L^2/\pi\mathcal{R}$ where $\mathcal{R}$ is the effective aspect ratio, and equating weight and lift at liftoff results in:

$$Z=Y\left\{\left[\left(\frac{C_{D,0}+C_{D,f}}{C_{D,i}}+1\right)\frac{C_L}{\pi\mathcal{R}}-\mu\right]\Big/(r-\mu)\right\}^{1/2} \tag{10}$$

with

$$Y=V/V_0$$

Estimates of the quantities in Eq. (10) are easily obtained using a textbook such as that by Nicolai.[2] With mechanical lift augmentation, the maximum C_L for transports is about 4 (Ref. 2, p. 2-15) on an effective aspect ratio of 5-6. Fighter aircraft achieve lower maximum values of C_L on aspect ratios of about 3 (Ref. 2, p. E-8). A reasonable approximation for $C_L/\pi\mathcal{R}$ is therefore about 0.2.

Under normal flight conditions, Nicolai (Ref. 2, p. 2-15) estimates the viscous drag to be 80% of the total with interference drag contributing about 4%. During takeoff, the landing gear adds drag but the lift coefficients are 5-10 times their cruise values and the induced drag instead of being 20% of the viscous drag will be 5-20 times that. It therefore is quite reasonable to estimate $(C_{D,0}+C_{D,f})/C_{D,i}$ at ⅛. This factor will be dominated by viscous effects so that a turbulent flow variation, $Y^{-0.2}$ should be used. With $\mu=0.025$ (Ref. 2, p.10-2)

$$Z=K_r f(Y)=K_r Y(1+0.143Y^{-0.2})^{1/2} \tag{11}$$

with

$$K_r=0.42/(r-0.025)^{1/2}$$

Table 1 Variation of f with the relative speed Y

Y	0.1	0.3	0.5	0.7	0.9
$f(Y)$	0.11	0.33	0.54	0.75	0.96

Table 2 Sensitivity functions for time and distance, G and H

Z	0.1	0.3	0.5	0.7	0.9	0.96
G	0.003	0.032	0.11	0.29	1.11	2.65
H	0.005	0.049	0.16	0.43	1.57	3.62

The thrust to weight ratio r is shown in Ref. 2 (pp. E-8, E-9) to range from about 0.2 for transports to 1.2 for modern fighters, so that K_r ranges from 0.4 to 1.0. Evaluations of $f(Y)$ are given in Table 1. The variable Z will therefore span the range 0,1 with the values for fighters tending to group near the lower bound. With this information, the sensitivity function can be evaluated in the range of Z of 0.1 to 0.96, as shown in Table 2. As liftoff is approached, the errors in time and distance estimates are thus shown to equal or exceed the errors incurred in the assumption of constant aerodynamic coefficients. Table 2 may be used to estimate the magnitudes of effective aerodynamic coefficients if simplified calculations were done in segments.

Estimates of Errors

With the aerodynamic estimates outlined above, it is possible to rewrite F as

$$F=0.175(1+0.143Y^{-0.2})\rho A V_0^2 C_L/2 \tag{12}$$

In the range of Y considered here, the variation ΔF due to viscous effects, at constant C_L, is about 7%. This translates into errors of time and distance between 0 and 18% and 0-26% for transports, and about 40% of those ranges for fighters. Most aircraft change their attitude significantly as they gather speed, and in fact, some fighter aircraft deliberately alter their attitude by mechanical means. Thus, even though the sensitivity of fighters is lower, their variations ΔF will be very high due to large changes in C_L during the ground run. For a moderate value of r the errors for a fighter could easily range up to 30% and 40% for estimates of time and distance, respectively. The errors for transports could be twice that. The procedure proposed by Powers is therefore not adequate for preliminary design calculations of generally accepted standards and should be viewed as a highly speculative estimate until accurate error estimates for particular situations are developed.

Concluding Remarks

At the present time, the utility of highly simplified analytical results is rather questionable in view of the ready availability of small computers which can easily be programmed to include realistic variations of principal parameters of the problem. Misguided attempts to develop simple analytical results cling to an antiquated tradition which must yield to advances in computers and the almost universal availability of computing equipment.

References

[1] Powers, S. A., "Critical Field Length Calculations for Preliminary Design," *Journal of Aircraft,* Vol. 18, Feb. 1981, pp. 103-107.

[2] Nicolai, L.M., *Fundamentals of Aircraft Design,* School of Engineering, University of Dayton, Dayton, Ohio, 1975.

AIAA 82-4057

Reply by Author to A. Wortman

S. A. Powers*
Vought Corporation, Dallas, Texas

DR. Wortman's interest in my paper is appreciated. However, his comments contain several errors in perception and in fact.

1) Most modern aircraft *do not* change their attitude as they "gather speed". This fact should be evident to anyone who has ridden a jet transport.

2) Aircraft do not accelerate/decelerate at high lift coefficients. Moderate flap deflections are used to maximize the rate-of-climb after takeoff. For one case presently in work, the lift coefficient during roll is approximately 0.3, with a $C_{L_{\max}}$ value of about 1.5 for a flap deflection of 15 deg.

3) The assumed value of the ratio of viscous plus profile drag to induced drag is far off the mark. For the case cited above, the value of their ratio is 64 instead of ⅛.

4) Evaluation of Wortman's Eq. (6b) indicates that 1.0 must be added to the tabulated values of G. Similarly, the tabulated quantities for H should all be negative.

5) Those of us who work in preliminary design are acutely aware of the fact that simple approximate methods which give answers now are highly preferable to more elaborate methods which promise answers "mañana".

Received Sept. 21, 1981.

*Senior Staff Engineer, Aircraft Aerodynamics. Associate Fellow AIAA.